FDTD Modeling of Transient Microwave Signals in Dispersive and Lossy Bi-Isotropic Media

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Abstract — **We present a novel Finite Difference Time Domain (FDTD) model of transient wave propagation in general dispersive bi-isotropic media with losses. The special properties of these materials may lead to new applications in microwave and millimeter-wave technology. While their frequency domain properties have been well described in the literature, their time domain behavior has only been modeled so far for special sub-classes and monochromatic time dependence. We have validated our method by first computing time-harmonic wave propagation through a bi-isotropic medium and comparing it with theoretical results. Agreement is typically better than one percent. Then we have computed transient field propagation in a general dispersive bi-isotropic medium.**

I. Introduction

In contrast to ordinary materials characterized by electric permittivity and magnetic permeability, bi-isotropic materials present two additional parameters in their constitutive equations, namely the *Tellegen* and *chirality* parameters, that relate the electric field ***E*** with the magnetic flux density ***B***, and the magnetic field ***H*** with the electric displacement ***D***. Electromagnetic waves in bi-isotropic media show the following interesting behavior [1]:

a) **Optical Rotatory Dispersion** causing a rotation of polarization;

b) **Circular Dichroism**, which modifies the nature of field polarization;

c) **Non-orthogonality** of electric and magnetic field vectors.

These properties have drawn considerable attention to bi-isotropic media due to their potential applications. Two subclasses of general bi-isotropic media are *Tellegen* and *chiral* media, in which only one of these two parameters is taken into account in the constitutive equations. Many attempts have been made to model bi-isotropic media in time domain [2]-[3], and good results have been obtained for special cases, such as chiral media or non-dispersive bi-isotropic media. However, no time domain formulation has been developed to date that models general bi-isotropic dispersive media with losses. In this paper, a full time-domain model of general bi-isotropic dispersive media is proposed. It is based on the FDTD technique, where the basic Yee cell has been modified to include the special relationships between the field vectors in bi-isotropic media. To validate our method we have computed the characteristic behavior of both monochromatic and transient electromagnetic waves traveling through a bi-isotropic medium and obtained with very good agreement with the theoretical results.

II. Time Domain Update Constitutive Equations

The constitutive equations for bi-isotropic media in frequency domain are given by [1]:

$$\begin{aligned}\vec{D}(\omega) &= \varepsilon\vec{E}(\omega) + \sqrt{\varepsilon_0\mu_0}\,(\chi - j\kappa(\omega))\vec{H}(\omega)\\ \vec{B}(\omega) &= \mu\vec{H}(\omega) + \sqrt{\varepsilon_0\mu_0}\,(\chi + j\kappa(\omega))\vec{E}(\omega)\end{aligned} \qquad (1)$$

where χ is the *Tellegen* parameter and $\kappa(\omega)$ is the *chirality* parameter. The frequency dependence of the *chirality* parameter is assumed to follow the *Condon* model [1]:

$$\kappa(\omega) = \frac{\tau\omega_o^{\,2}\omega}{\omega_o^{\,2} - \omega^2 + j2\omega_o\xi\omega} \qquad (2)$$

where ω_0, is a characteristic resonant frequency, τ a time constant and ξ the damping factor.

In order to obtain a time domain expression for the chirality parameter, the imaginary unit that appears in the constitutive equations (1) is introduced in the chirality parameter expression (2), and the time-dependent chirality parameter is obtained by *inverse Laplace transform*:

$$L^{-1}(\kappa'(\omega)) = \kappa'(t) = \frac{\tau\omega_o^2 e^{-\xi\omega_0 t}}{\sqrt{1-\xi^2}} sin\left(\phi - \omega_0\sqrt{1-\xi^2}\,t\right) \qquad (3)$$

where $\kappa'(\omega)$ is defined as $\kappa'(\omega) = j\,\kappa(\omega)$ and the angle ϕ is defined as $\phi = acos(\xi)$.

In time domain the relationship given in eq. (1) becomes a convolution. If we discretize these equations and make

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the approximation that all the field quantities are constant over each discrete time interval, and if we assume that all fields are zero for $t<0$, then the integration becomes, in part, a summation:

$$\vec{D}(n) = \varepsilon\vec{E}(n) + \frac{\chi}{c_0}\vec{H}(n) - \frac{1}{c_0}\sum_{m=0}^{n-1}\vec{H}(n-m)\int_{m\Delta t}^{(m+1)\Delta t}\kappa'(\tau)d\tau$$
$$\vec{B}(n) = \mu\vec{H}(n) + \frac{\chi}{c_0}\vec{E}(n) + \frac{1}{c_0}\sum_{m=0}^{n-1}\vec{E}(n-m)\int_{m\Delta t}^{(m+1)\Delta t}\kappa'(\tau)d\tau \quad (4)$$

The form of the *chirality* parameter $\kappa'(t)$ does not allow recursive updating of the discrete convolution. However, following the procedure of Luebbers and Hunsberger [4], we define a complex time domain chirality:

$$\hat{\kappa}'(t) = -j\frac{\tau\omega_0^2}{\sqrt{1-\xi^2}}e^{\left(-\xi\omega_0 t + j(\phi - \omega_0\sqrt{1-\xi^2}t)\right)} \quad (5)$$

$$\kappa'(t) = Re(\hat{\kappa}(t)) \quad (6)$$

The exponential form of the complex *chirality* now makes it suitable for recursive convolution. Details are found in [4] and will not be repeated here. We obtain the following final time domain constitutive update equations:

$$\vec{D}(n) = \varepsilon\vec{E}(n) + \frac{\chi}{c_0}\vec{H}(n) - \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]\vec{H}(n) + Re\left[{}^{H}\hat{\vec{\psi}}(n-1)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right]$$
$$\vec{B}(n) = \mu\vec{H}(n) + \frac{\chi}{c_0}\vec{E}(n) + \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]\vec{E}(n) + Re\left[{}^{E}\hat{\vec{\psi}}(n-1)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right] \quad (7)$$

where ${}^{E}\hat{\vec{\psi}}(n)$ denotes the complex convolution summation of the electric field with the complex chiral response of the material to an impulse, at the instant $n\Delta t$.

III. New FDTD Formulation

In order to model bi-isotropic media, we have modified the traditional FDTD method. Although we present in this paper only the implementation of a 1-D mesh and algorithm, this formulation can be extended to the 2-D and 3-D cases as well. We assume uniform wave propagation in the z-direction. In order to capture the rotation of the fields (caused by the *chirality* parameter) and their non-orthogonality (caused by the *Tellegen* parameter) in the transversal plane, we model the x- and y-components of both the electric and magnetic fields.

The peculiar constitutive equations of bi-isotropic media that relate the electric and magnetic fields in the same point and at the same time instant, require a modification of the Yee cell and the traditional FDTD algorithm. Our new cell includes four quantities in each node, namely ***E***, ***D***, ***H*** and ***B***, related by the constitutive equations, and we have two different kinds of nodes, the x-nodes where we define the x-components of the fields (E_x, D_x, H_x, B_x), and the y-nodes with the y-components (E_y, D_y, H_y, B_y). Fig. 1 shows the x- and y-nodes staggered in space and time.

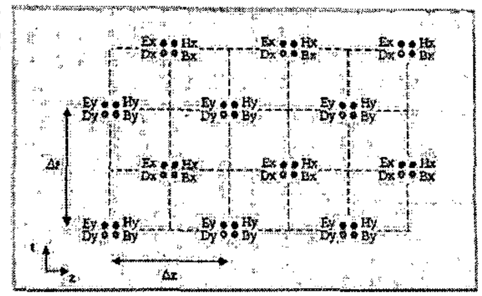

Fig. 1. Modified FDTD mesh for bi-isotropic media

We have introduced losses in the algorithm by using the perturbation method for small losses. First, we update the x-components of ***D*** and ***B*** everywhere in the mesh by means of the lossless standard FDTD update equations, then, within the same time step, the x-components of ***E*** and ***H*** are calculated using the x-component of the vector constitutive equations in time domain (7) derived above:

$$D_i^x(n) = \varepsilon E_i^x(n) + \frac{\chi}{c_0}H_i^x(n) - \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]H_i^x + Re\left[{}^{H}\hat{\vec{\psi}}_i^x(n-1)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right]$$
$$B_i^x(n) = \mu H_i^x(n) + \frac{\chi}{c_0}E_i^x(n) + \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]E_i^x + Re\left[{}^{E}\hat{\vec{\psi}}_i^x(n-1)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right] \quad (8)$$

Once we know E_x and H_x in the absence of losses, we recalculate D_x and B_x from the time average value of each field, using again standard FDTD equations, this time accounting for losses by considering small electric and magnetic conductivities. These must be small enough to justify the assumption that E_x and H_x do not change significantly from their values in the lossless case. Finally we compute E_x and H_x for the case of small losses using the x-component of the constitutive equations given in (8). At one half time step later, the y-components of ***D*** and ***B*** are computed using the same standard FDTD update equations in the absence of losses, at the same time step, we calculate the y-components of ***E*** and ***H*** using the y-component of the vector constitutive equations that relates the quantities in our y-node:

$$D_{i+1/2}^y(n+1/2) = \varepsilon E_{i+1/2}^y(n+1/2) + \frac{\chi}{c_0}H_{i+1/2}^y(n+1/2) - \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]H_{i+1/2}^y(n+1/2) + Re\left[{}^{H}\hat{\vec{\psi}}_{i+1/2}^y(n-1/2)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right]$$
$$B_{i+1/2}^y(n+1/2) = \mu H_{i+1/2}^y(n+1/2) + \frac{\chi}{c_0}E_{i+1/2}^y(n+1/2) + \frac{1}{c_0}\left[Re\left[\hat{\kappa}'(0)\right]E_{i+1/2}^y(n+1/2) + Re\left[{}^{E}\hat{\vec{\psi}}_{i+1/2}^y(n-1/2)e^{-\omega_0(\xi+j\sqrt{1-\xi^2})\Delta t}\right]\right] \quad (9)$$

As we did in the case of the x-components, once we know E_y and H_y in the absence of losses, we recalculate D_y and B_y from the time-average value of each field, using again standard FDTD equations and accounting for losses due to small electric and magnetic conductivities. Finally we compute E_y and H_y for the lossy case using the *y*-component of the vector constitutive equations (9).

IV. RESULTS

To validate our formulation we have first computed the characteristic behavior of a monochromatic electromagnetic wave traveling through a bi-isotropic medium, so that we could compare our results with frequency domain theoretical behavior of waves in bi-isotropic media [1]. After this preliminary validation, we have performed numerical pulse propagation experiments and modeled the first transient wave propagation in such a medium.

A. Validation of Monochromatic Wave Propagation in Bi-Isotropic Media

As mentioned in the introduction, electromagnetic waves in bi-isotropic media exhibit the following properties [1]:

Optical Rotatory Dispersion causes a rotation of polarization due to different phase velocities of the right- and left-handed circularly polarized waves. The angle of rotation depends on the real part of the chirality parameter.

Circular Dichroism modifies polarization by introducing ellipticity. It is due to the different absorption coefficients of the right- and left-handed circularly polarized waves. Depends on the imaginary part of the chirality.

Non-orthogonality of electric and magnetic field vectors due to the non-zero *Tellegen* parameter.

We performed all simulations in a 1-D computational domain that was 10,000 cells (Δz = 1/3 mm) long, and applied the excitation at the point 4000 Δz. The mesh boundaries were remote enough to avoid possible reflections.

In the first simulation the parameters of the medium were: $\mu_r = 1$, $\varepsilon_r = 2$, $\chi = 0$, $\tau = 4\ ps$, $\omega_0 = 2\pi\,10^9\ rad/s$. To allow comparison with analytical frequency domain results we injected a time-harmonic electric field ($f = 3\ GHz$) linearly polarized at *45 degrees* with respect to the *x*- and *y*-axes. Both the lossless and the lossy case were considered. At *3 GHz*, and in the absence of losses ($\xi=0$), the value of the *chirality* is $\kappa = -9.42\cdot 10\text{-}3+j0$. Since the imaginary part of the *chirality* is zero, the wave preserves its linear polarization, but due to the negative value of the real part of the *chirality* parameter, the polarization rotates clockwise when looking in the direction of propagation (*+z direction*). The rotatory property of the medium for the *lossless* case is shown in Fig 2. In the *lossy* case ($\xi=0$, $\sigma_e=1.5\ 10^{-3}\ S/m$, $\sigma_m=1.5\ 10^{-3}\ \Omega/m$) at *3 GHz*, the value of the *chirality* is $\kappa = -8.97\cdot 10^{-3}\text{-}j2.018\,10^{-3}$. Due to the non-zero imaginary part of the chirality the linear polarization of the fields degenerates into elliptical polarization as they propagate (dichroism). Since $Re(\kappa)<0$ the polarization direction rotates clockwise. The rotatory property and circular dichroism of the medium are visualized in Fig. 3.

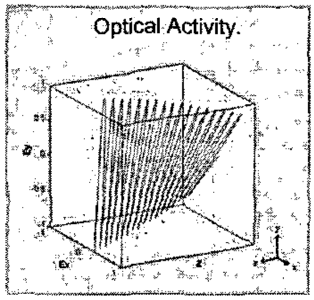

Fig. 2. Polarization of the electric field at 3000 *ps*. The direction of polarization rotates clockwise as the wave propagates through the lossless bi-isotropic medium.

The theoretical and computed values of the rotation angle at $1500\Delta z$ and $3000\Delta z$ (Δz=1/3 mm) are compared in Table 1

TABLE 1

Theoretical and simulated angles of rotation of the polarization.

Distance to the Source	Theoretical Angle	Computed Angle	Error in Percent
1500Δz	16.965	17.127	+0.95
3000Δz	33.929	34.158	+0.67

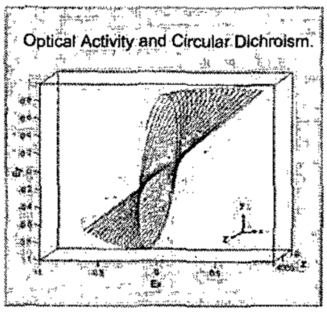

Fig. 3. Rotation of the polarization and circular dichroism at 3000 *ps* when a wave propagates in a lossy bi-isotropic medium.

In a bi-isotropic medium the angle between ***E*** and ***H*** is determined by the *Tellegen* parameter χ. We have computed it for a medium with parameters $\mu_r = 1$, $\varepsilon_r = 2$, $\tau = 3\ ps$, $\omega_0 = 2\pi\,10^9\ rad/s$ and $\xi=0$. Three different values of χ were considered $\chi=0.1$, $\chi=0.2$, and $\chi=0.3$. The excitation was the same as in the previous simulation. Table 2 shows the theoretical and simulated values of this angle.

Fig. 4 visualizes $\boldsymbol{E}$ and $\boldsymbol{H}$ (multiplied by Z_c) in the case of $\chi = 0.3$ at the points $1\Delta z$, and $3000\ \Delta z$ from the source, this figure demonstrates simultaneously the three basic properties of waves propagating in general bi-isotropic media, namely rotation of the polarization, circular dichroism and non-orthogonality of the electric and magnetic fields.

TABLE 2

Theoretical and simulated angles between $\boldsymbol{E}$ and $\boldsymbol{H}$.

Tellegen Parameter	Theoretical Angle	Computed Angle	Relative Error in Percent
χ=0.1	94.055	93.979	-0.08
χ=0.2	98.130	97.985	+0.14
χ=0.3	102.247	102.38	+0.12

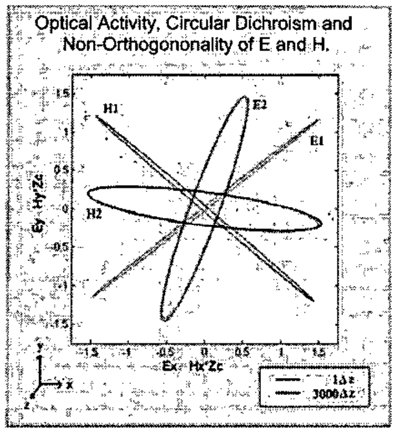

Fig. 4. Rotation of the polarization, circular dichroism and non-orthogonality of $\boldsymbol{E}$ and $\boldsymbol{H}$ at *3000 ps*.

B. Transient Field Propagation in a General Dispersive Bi-Isotropic Medium.

In the previous section we have validated the proposed algorithm by comparing simulation results for monochromatic waves with frequency domain theoretical behavior of waves in bi-isotropic media [1]. In order to demonstrate the full transient capability of this time domain approach, we have computed the propagation of high-frequency pulses in a *general dispersive bi-isotropic medium.*

In this simulation the 1-D computational domain was the same than in the previous case. The bi-isotropic medium had the following parameters: $\mu_r = 1$, $\varepsilon_r = 2$, $\chi = 0$, $\tau = 4\ ps$, $\omega_0 = 2\pi\ 10^9\ rad/s$ and $\xi=0$, the conductivities σ_e *and* σ_m were set to zero. At the point $4000\ \Delta z$ we injected a group of three band-limited pulses with a center frequency of *9 GHz,* linearly polarized in *x*-direction. For all frequencies within the spectrum of the pulses the real part of the chirality is negative; therefore, the polarization rotates clockwise. The rotation of the polarization of the fields, extracted at *3000 ps,* is shown in Fig. 5.

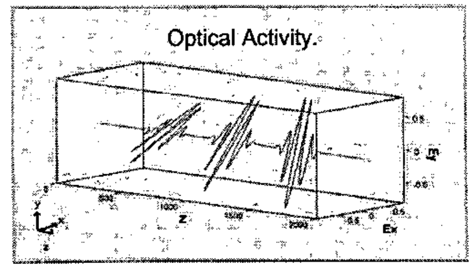

Fig. 5. Polarization of the electric field at 3000 *ps*, it rotates clockwise as the wave propagates through the medium.

V. CONCLUSION

We propose a novel time domain model of wave propagation in general dispersive bi-isotropic media, formulating the constitutive relationships by recursive convolution. The traditional FDTD method and Yee cell have been modified. Our new formulation involves updating electric and magnetic fields in the same point and at the same time step. While this model has been implemented and validated here for the 1-D case only, it can be extended to the two- and three-dimensional cases. To our knowledge this is the first time domain formulation that allows full transient modeling of general dispersive bi-isotropic media, including losses.

A series of numerical experiments have demonstrated the validity and accuracy of the proposed algorithm. Simulated rotation angles agree with theoretical values within typically less than one percent. Finally, numerical experiments of transient fields in a general dispersive bi-isotropic medium have been performed to demonstrate its full transient capability.

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